Robust-Fuzzy-Probabilistic Optimization for a Resilient, Sustainable Supply Chain with an Inventory Management Approach by the Seller

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Abstract

The current paper deals with modeling a resilient, sustainable supply chain with an inventory management approach by the seller under the uncertainty of demand and system costs. The importance of inventory management by the seller in the sustainable supply chain has led them to consider a set of buyers and sellers whose goal is to minimize the total costs of ordering, shortage, maintenance, and use of the vehicle to make the right decisions to fulfill the orders. Due to the indeterminacy of the model parameters, the robust-fuzzy-probabilistic optimization method has been used. The calculation results with the invasive weed optimization algorithm and the Baron method show that with the increase in the uncertainty rate in the network, the amount of demand has increased. Therefore, the total ordering, maintenance, and shortage costs have increased. Also, with the increase of the stability coefficient of the model, the total cost of inventory management by the seller has increased, and a greater amount of customer demand has been estimated. On the other hand, with the increase in resilience, the amount of orders transferred to the buyer has decreased. Also, the calculation results show the high efficiency of the invasive weed optimization algorithm in solving the resilient, sustainable supply chain model with the seller's inventory management approach.

Keywords: Resilient, Sustainable supply chain, Inventory management by the seller, Robust-fuzzy-probabilistic optimization, Invasive weed optimization algorithm.

1 | Introduction

Inventory management by the seller is a logistics strategy in which the supplier can control and manage its customers' inventory through the information they provide. In today's competitive world, organizations want to reduce costs and inventory levels and achieve maximum profit. Inventory management by the seller plays an essential role in achieving these goals. By implementing this system, the seller decides to replenish the

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inventory on behalf of the buyer's organization [1]. The difference between the model of inventory management by the seller and other models is that in this system, the retailer, in addition to sharing detailed information about his inventory level and customer demand with the supplier, leaves all the decisions about inventory management to the supplier. The retailer's benefits of vendor inventory management include reducing overhead costs and shifting inventory costs to the supplier [2]. The benefits of using inventory management by the seller for the supplier are not created clearly and directly. With the seller's long-term use of an inventory management system and the establishment of communication in shipments between the supplier and several retailers, inventory costs can be reduced [3].

In recent years, one of the best strategies used by companies to manage and reduce inventory costs is vendor-managed inventory. The seller manages and controls inventory throughout the supply chain in this policy. Many retailers, such as Wal-Mart, use this strategy for their suppliers. This strategy can help managers improve their product inventory agility. This strategy enables organizations to share information between vendors (suppliers) and retailers. When information sharing is implemented properly between sellers and retailers, sellers are aware of inventory levels at retailers [4].

Given the importance of the seller's inventory management system, this study presents a model for a resilient, sustainable supply chain under uncertainty. Nowadays, the economic aspects are not the only consideration for the managers of the production units, and the environmental and social aspects are equally important. The seller's various decisions in the inventory management system include determining the optimal amount of the economic order and the allowed shortage, as well as determining the optimal number of vehicles to transport the products. To achieve the correct decisions, the objective function of minimizing the total costs has been considered while respecting the limits of stability and resilience. On the other hand, uncertainty in the amount of demand and failure to determine its exact amount has led managers to face difficulties in making production and sales decisions. Therefore, in this paper, the robust-fuzzy-probabilistic optimization method controls the model's non-deterministic parameters, including demand and system costs.

In the following six sections, the importance of modeling the problem of inventory management by the seller is discussed first. The research literature is reviewed in the second part, and the resulting research gaps are identified. Then, a model of a resilient, sustainable supply chain with an inventory management approach by the seller is introduced, and the robust-fuzzy-probabilistic optimization method is used to control the model's parameters. In the fourth section, the design of the initial solution and the introduction of the invasive weed optimization algorithm are expressed. The fifth part discusses the analysis of different numerical examples and the sensitivity analysis of the problem. Finally, the conclusions and future suggestions of the research are given.

2|Literature Review

Inventory management by the seller is a business type in which the buyer of a product provides detailed information about that product to the seller, and the seller must pay for the sale and the acceptable inventory of materials in the warehouse, which is usually at the place of consumption of the buyers (usually a shop). As a two-way relationship, VMI reduces the likelihood that a business's production will fall asleep, goods will remain in stock, and production will be reduced in the production process. This issue has been studied by many researchers in the supply chain network, and the most important of these researches are discussed in the following.

Karbasi Bonab et al. [5] investigated a dual-objective vendor inventory management model with fuzzy demand for a supply chain problem with multiple vendors and retailers. Their goal was to minimize the total inventory cost and optimize warehouse space. NSGA II algorithm was used to solve the two-objective model. Mohammadzadeh and Mirzazadeh [6] presented an inventory-production model under inventory management policy by the seller in fuzzy conditions. In this model, they specified decisions such as retail price. Teng et al. [7] modeled an integrated transportation inventory problem in a vendor-driven inventory management system. This problem aims to minimize the logistics cost in the distribution network, including

inventory cost, distribution cost, and time penalty cost. To solve their model, they used two algorithms to simulate refrigeration and an ant colony. Ganesh Kumar and Uthayakumar [8] modeled inventory management policies by the seller under environmental conditions. Their goal was to reduce the costs of the entire system, including the costs of fines and taxes based on greenhouse gas emissions. They used a genetic algorithm to solve the problem. The results show that the system performs better when operated under unequal shipping policies and vendor-managed inventory agreements. Gharaei et al. [9] presented an inventory management model by a vendor in a supply chain with multiple buyers and multiple products in contingencies. The objective is to determine the minimum-cost optimal batch size policy in an integrated supply chain that finds the number of vendor batches for each product shipped and the volume of batches shipped to buyers to minimize total cost. They used the incremental penalty algorithm to solve the problem. The results of optimal criteria obtained in numerical examples and sensitivity analysis show the excellent performance of the method used.

Karimian et al. [10] proposed a quantitative multi-product economic production model considering shortages for single-vendor and multi-retailer supply chains under vendor inventory management policy. They considered the parameters of the model to be random. In this paper, the geometric programming approach is used to find the optimal solution to the nonlinear stochastic programming problem to minimize the mean variance of the total inventory cost of the system. Huynh and Yenradee [11] developed a vendor inventory management model for a supply chain that included multiple vendors and manufacturers. They wanted to minimize the total inventory management cost and choose the right trucks to transport the items. For this, they chose the genetic algorithm to solve the problem. Dai et al. [12] presented an inventory-routing model for perishable products in a vendor-driven inventory management supply chain. The model they presented included a manufacturer and several retailers. The main goal of the models they presented was to minimize the costs of the entire supply chain network. Cuckoo and Clark-Wright algorithms have been used to solve the problem. The results show that the proposed algorithm performs better than the optimization solver. In addition, the results show that the average total cost may be reduced using the warehousing strategy in some cases.

Ashraf et al. [13] propose a type-2 fuzzy vendor inventory management system, where type-2 interval fuzzy numbers represent demand and order quantity. The proposed model aims to minimize the total cost for a single-vendor-retailer business, a multi-product commodity, and a centralized warehouse of retailer-managed inventory. Since the proposed model is NP-hard, a solution approach based on particle swarm optimization is developed to solve it properly. Karimi et al. [14] presented a Stackelberg model for optimizing price and service level inventory decisions in a vendor-driven inventory management system.

Poursoltan et al. [15] modeled the two-level closed-loop supply chain problem under an inventory management contract by vendor and learning effects. Due to the Np-Hard nature of their model, meta-heuristic algorithms have been proposed to solve the problem. The results show the high efficiency of these algorithms in solving different numerical examples. Asadkhani et al. [16] presented an inventory management model by the vendor, considering quality requirements and environmental issues. In this model, the main goal is to provide a decision support model to find the optimal harvesting policy as a joint quality policy. They used the analysis algorithm to achieve the optimal solution. The sensitivity analysis shows that the fraction of incomplete items greatly affects the optimal withdrawal policy.

Lotfi et al. [17] modeled an inventory management model in a sustainable supply chain network. In this article, they considered aspects of sustainability such as social, environmental, and economic. To solve the problem, they used the exact method. They concluded that the total cost of inventory management by the seller is lower than the total cost without considering this system. Also, they imagined some parameters of the problem as non-deterministic.

Astanti et al. [18] presented a supply chain inventory model to help managers make optimal inventory decisions considering logistics costs and carbon emissions. They also considered the effects of product failure and quality problems. Their objective was to minimize the total costs considering the carbon price.

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According to the literature review, most of the articles have addressed the seller's modeling of the inventory management system in deterministic conditions, and fewer articles have used new uncertainty optimization methods. Also, the seller's consideration of stability and resilience in the supply chain network for the inventory management system has not been fully and comprehensively studied. Therefore, in this article, considering the research gaps, the researchers have presented a comprehensive model of the inventory management system by the vendor for the resilient, sustainable supply chain under uncertainty and using the robust-fuzzy-probabilistic optimization method to control the cost parameters and have used demand. Additionally, an invasive weed optimization algorithm is proposed to solve the model.

3|Problem Definition

In this paper, an inventory management system by the vendor is modeled for a resilient, sustainable supply chain. In the stable supply chain network, according to *Fig. 1*, there is a set of sellers ($i \in I$) and buyers ($j \in J$). Buyers have uncertain demand for different types of products ($p \in P$) in different scenarios ($s \in S$). They send their orders of various products to the sellers. Based on the limitations in the system, such as the limitation of greenhouse gas emission, the limitation of energy consumption, and the limitation of job creation, the sellers provide some of the buyers' demands. The model tries to make order resilience flexible in this supply chain network. The estimated demand quantity of buyers should be sent by heterogeneous vehicles ($r \in R$). This model's main goal is to minimize the total cost of ordering, maintaining, shortage, and using the vehicle in different scenarios. Therefore, the sustainability of the supply chain network has been examined in three areas: economic, social, and environmental.
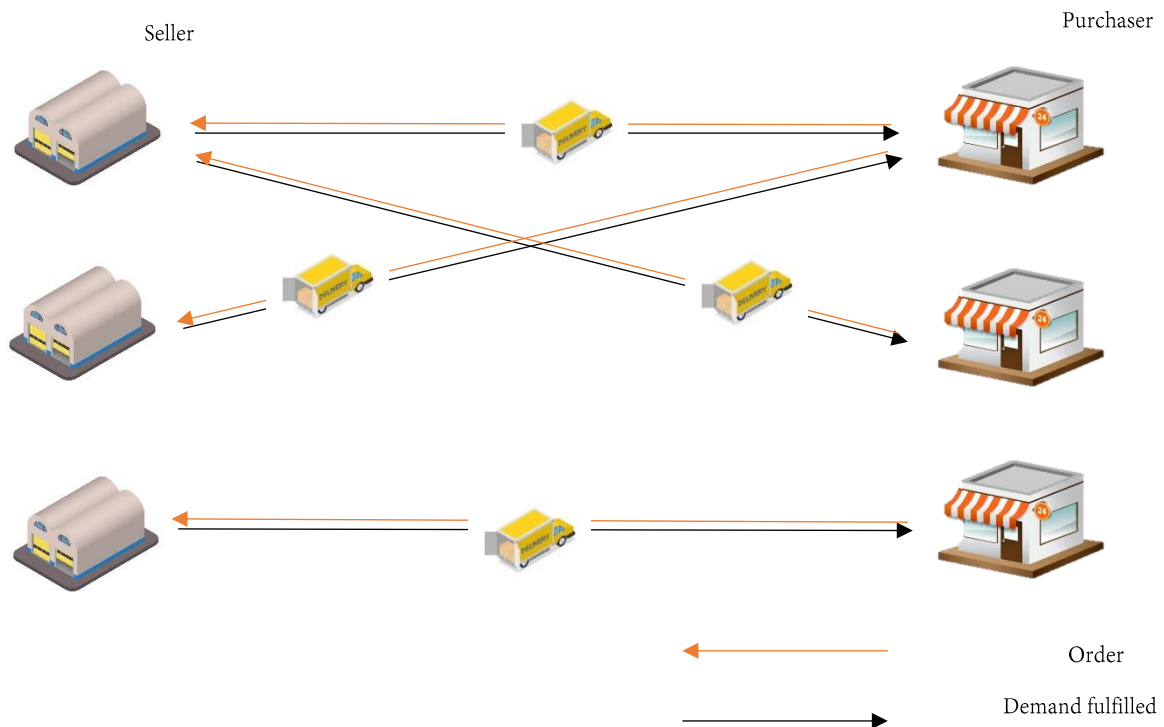

Fig. 1. Resilient sustainable supply chain network.

The assumptions of the mathematical model are as follows:

    I.   The mathematical model is multi-product and single-period.

    II.   Demand and costs related to the supply chain network are considered non-deterministic.

    III.   Lack of product demand is allowed.

    IV.   Order delivery time is considered instantaneous.

V.   The time horizon is infinite.

Due to the indeterminacy of demand parameters and network costs, each parameter in different scenarios is known as a triangular fuzzy number. Therefore, the robust-fuzzy-probabilistic optimization method has been used to control non-deterministic parameters. The reason for using the stable-fuzzy-probabilistic optimization method is the high efficiency of this method compared to any of the traditional methods in the literature. With the definition of the above problem and the assumptions of the problem, the following symbols are presented for modeling:

Parameters

fi_r	The cost of using the vehicle $r \in R$.
dem_{jps}	Buyer demand $j \in J$ of product $p \in P$ in scenario $s \in S$.
ov_{ip}	Cost of ordering product $p \in P$ at seller $i \in I$.
ob_{jp}	Cost of ordering product $p \in P$ in buyer $j \in J$.
p_s	The probability of scenario $s \in S$.
f_p	Space required for product $p \in P$.
h_{jp}	The cost of keeping product $p \in P$ in buyer $j \in J$.
s_{ip}	Shortage penalty cost at seller $i \in I$ for product $p \in P$ in each period.
k_{ip}	Time-independent shortage penalty cost at seller $i \in I$ for product $p \in P$.
e_i	The amount of greenhouse gas produced in seller $i \in I$.
φ_{is}	The maximum allowed amount of greenhouse gas emitted in seller $i \in I$ in scenario $s \in S$.
t_i	The amount of energy consumed by seller $i \in I$.
δ_{is}	The maximum allowed amount of energy consumed in seller $i \in I$ in scenario $s \in S$.
v_i	Number of jobs created in seller $i \in I$.
ρ_{is}	Minimum allowed number of jobs created in vendor $i \in I$ in scenario $s \in S$.
V	The number of jobs created per order.
T	The amount of energy consumed per order.
E	The amount of greenhouse gas produced per order.
F_r	Total available space of vehicle $r \in R$ to transport all products.
N	Total number of orders for all products.
γ	Resilience factor in the order.
θ	Shortage-to-order ratio.
α_s	Uncertainty rate in fuzzy number levels in scenario $s \in S$.
ω	Coefficient of influence of stability of the model.

Decision variables

Q_{ijps}	Order amount transferred from seller $i \in I$ to buyer $j \in J$ for product $p \in P$ in scenario $s \in S$.

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D_{ijps}	Allocated demand of buyer $j \in J$ to seller $i \in I$ for product $p \in P$ in scenario $s \in S$.
B_{ijps}	Allowable shortage from seller $i \in I$ to buyer $j \in J$ for product $p \in P$ in scenario $s \in S$.
U_{rs}	Number of vehicles $r \in R$ used in scenario $s \in S$.

The mixed integer nonlinear mathematical programming model in the resilient, sustainable supply chain network with an inventory management approach by the seller is designed as follows

$$\text{Min } Z = E[Z] + (E[Z] - Zmin)$$

$$+ \sum_{s=1}^{S} p_s(E[Z] - E[Z_s]) + \sum_{j=1}^{J} \sum_{p=1}^{P} \sum_{s=1}^{S} p_s \left(dem_{jps}^3 - \frac{(\alpha_s - \omega)dem_{jps}^3 + (1 - \alpha_s)dem_{jps}^2}{1 - \omega} \right), \quad (1)$$

s.t.

$$\frac{D_{ijps}}{Q_{ijps}} \le N\gamma, \qquad \text{for all } i \in I. j \in J. p \in P. s \in S, \tag{2}$$

$$f_p Q_{ijps} \le \sum_{r=1}^{R} U_{rs} F_r, \qquad \text{for all } i \in I. j \in J. p \in P. s \in S, \tag{3}$$

$$B_{ijps} \le \theta Q_{ijps}, \qquad \text{for all } i \in I. j \in J. p \in P. s \in S, \tag{4}$$

$$D_{ijps} = \frac{\widetilde{dem}_{jps}}{|I|}, \qquad \text{for all } i \in I. j \in J. p \in P. s \in S, \tag{5}$$

$$e_i + E \sum_{j=1}^{J} \sum_{p=1}^{P} Q_{ijps} \le \varphi_{is}, \qquad \text{for all } i \in I. s \in S, \tag{6}$$

$$t_i + T \sum_{j=1}^{J} \sum_{p=1}^{P} Q_{ijps} \le \delta_{is}, \qquad \text{for all } i \in I. s \in S, \tag{7}$$

$$v_i + V \sum_{j=1}^{J} \sum_{p=1}^{P} Q_{ijps} \ge \rho_{is}, \qquad \text{for all } i \in I. s \in S, \tag{8}$$

$$E[Z] = \sum_{s=1}^{S} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{p=1}^{P} p_s \left(\frac{\left(\left(\frac{ov_{ip}^1 + ov_{ip}^2 + ov_{ip}^3}{3} \right) + \left(\frac{ob_{jp}^1 + ob_{jp}^2 + ob_{jp}^3}{3} \right) \right) D_{ijps}}{Q_{ijps}} \right)$$

$$+ \sum_{s=1}^{S} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{p=1}^{P} p_s \left(\frac{\left(\frac{h_{jp}^1 + h_{jp}^2 + h_{jp}^3}{3} \right) (Q_{ijps} - B_{ijps})^2}{2Q_{ijps}} \right) \tag{9}$$

$$+ \sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_s \left(\frac{\left(\dfrac{s_{ip}^1 + s_{ip}^2 + s_{ip}^3}{3}\right) B_{ijps}}{2Q_{ijps}} + \frac{\left(\dfrac{k_{ip}^1 + k_{ip}^2 + k_{ip}^3}{3}\right) B_{ijps} D_{ijps}}{Q_{ijps}} \right)$$

$$+ \sum_{r=1}^{R}\sum_{s=1}^{S} p_s fi_r U_{rs}$$

$$E[Z_s] = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_s \left(\frac{\left(\left(\dfrac{1-\omega}{2}\right)\left(\dfrac{ov_{ip}^1 + ov_{ip}^2}{3}\right) + \left(\dfrac{\omega}{2}\right)\left(\dfrac{ov_{ip}^2 + ov_{ip}^3}{3}\right)\right) D_{ijps}}{Q_{ijps}} \right)$$

$$+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_s \left(\frac{\left(\left(\dfrac{1-\omega}{2}\right)\left(\dfrac{ob_{jp}^1 + ob_{jp}^2}{3}\right) + \left(\dfrac{\omega}{2}\right)\left(\dfrac{ob_{jp}^2 + ob_{jp}^3}{3}\right)\right) D_{ijps}}{Q_{ijps}} \right)$$

$$+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_s \left(\frac{\left(\left(\dfrac{1-\omega}{2}\right)\left(\dfrac{h_{jp}^1 + h_{jp}^2}{3}\right) + \left(\dfrac{\omega}{2}\right)\left(\dfrac{h_{jp}^2 + h_{jp}^3}{3}\right)\right)\left(Q_{ijps} - B_{ijps}\right)^2}{2Q_{ijps}} \right)$$

(10)

$$+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_s \left(\frac{\left(\left(\dfrac{1-\omega}{2}\right)\left(\dfrac{s_{ip}^1 + s_{ip}^2}{3}\right) + \left(\dfrac{\omega}{2}\right)\left(\dfrac{s_{ip}^2 + s_{ip}^3}{3}\right)\right) B_{ijps}}{2Q_{ijps}} \right)$$

$$+ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_s \left(\frac{\left(\left(\dfrac{1-\omega}{2}\right)\left(\dfrac{k_{ip}^1 + k_{ip}^2}{3}\right) + \left(\dfrac{\omega}{2}\right)\left(\dfrac{k_{ip}^2 + k_{ip}^3}{3}\right)\right) B_{ijps} D_{ijps}}{Q_{ijps}} \right)$$

$$+ \sum_{r=1}^{R} p_s fi_r U_{rs}, \qquad \text{for all } s \in S,$$

$$Zmin = \sum_{s=1}^{S}\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P} p_s \left(\frac{(ov_{ip}^1 + ob_{jp}^1) D_{ijps}}{Q_{ijps}} + \frac{h_{jp}^1 \left(Q_{ijps} - B_{ijps}\right)^2}{2Q_{ijps}} + \frac{s_{ip}^1 B_{ijps}}{2Q_{ijps}} \right.$$

(10)

$$\left. + \frac{k_{ip}^1 B_{ijps} D_{ijps}}{Q_{ijps}} \right) + \sum_{r=1}^{R}\sum_{s=1}^{S} p_s fi_r U_{rs},$$

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$$Q_{ijps}; D_{ijps}; B_{ijps} \geq 0, \tag{12}$$

$$U_{rs} \geq 0 \text{ and integer.} \tag{13}$$

Eq. (1) shows the total costs of ordering the product, shortage, maintenance, and vehicle use. *Eq. (2)* states that the number of orders for the seller to the buyer should be less than the total number of orders considering resilience. *Eq. (3)* ensures that the number of transferred products, considering the space of each product, does not exceed the total available space. *Eq. (4)* guarantees that the seller's shortage to the buyer should not exceed its ratio to the order amount. *Eq. (5)* calculates the average demand allocated to each buyer by the seller. *Eq. (6)* limits the amount of greenhouse gas each vendor produces for each scenario. *Eq. (7)* limits the energy each seller consumes for each scenario. *Eq. (8)* guarantees that the total number of jobs created due to the system design should be greater than the minimum number of jobs created. *Eq. (9)* shows the average costs of the entire inventory management system. *Eq. (10)* shows the average costs of model stability in each scenario. *Eq. (11)* calculates the minimum stability costs of the model. *Eqs. (12)* and *(13)* show the type of decision-making variables.

The model designed in this section is a nonlinear mathematical programming model of various integers placed in inventory and allocation models. Considering the NP-Hardness of the transportation and allocation models in the resilient, sustainable supply chain network, it can be said that the degree of difficulty of the designed model is at least equal to the degree of difficulty of the transportation and allocation models. Therefore, the designed model is also among NP-hard problems.

This paper uses an invasive weed optimization algorithm to solve the problem. In the next section, after examining the solution method, the design of the initial solution to solve the problem is discussed.

4|Solution Method

4.1|Invasive Weed Optimization Algorithm

Invasive weeds are found in almost all man-made fields and gardens, and regardless of how much and how hard we try to eradicate them, they almost always win. Studying the behavior of these species and learning from their reproduction, survival, and adaptability can be informative for humans. The weed optimization algorithm is an intelligent and evolutionary optimization algorithm inspired by the reproduction, survival, and adaptability of invasive weeds [19].

Invasive weed is a phenomenon that searches for optimality and finds the best environment for life. It quickly adapts itself to environmental conditions and is resistant to changes. At first, an invasive weed reproduces to increase the quantity and cover the available environment (searching behavior), but due to capacity limitations, it continues to grow competitively by increasing the quality (greedy behavior).

In general, the purpose of invasive weeds is to find the best environment for life. The initial population (a certain number of seeds) is produced and dispersed in the first stage. In the second stage, after growing and becoming a plant, the dispersed seeds produce seeds by themselves according to their suitability. In the third stage, seedlings are scattered around the parent and grow. Finally, the second and third stages are repeated until the population does not exceed a certain limit (available range); otherwise, the plants with better competence remain among the available plants, and the rest are lost.

The steps of the invasive weed optimization algorithm are as follows [20]:

   I.   Generate a population of initial responses.

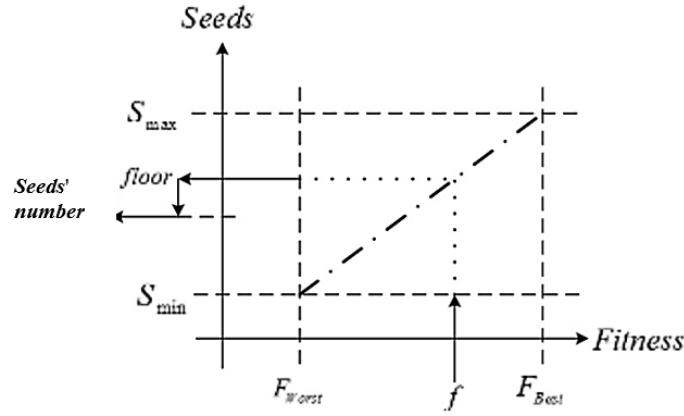 II.   Seed propagation based on the degree of fitness according to *Fig. 2.*

Fig. 2. Seed propagation based on fitness.

The method of calculating the number of seeds that can be produced by a plant based on the merit of the plant is according to *Eq. (14)*

$$s = \left[s_{min} + (S_{max} - S_{min}) * \frac{f - f_{worst}}{f_{best} - f_{worst}} \right]. \tag{14}$$

III.   The offspring seeds are scattered around the parent plant with a normal distribution $\Delta x_i \sim N(0. \sigma_t^2)$.

The standard deviation in this distribution is reduced over time according to *Fig. 3* and calculated this way.
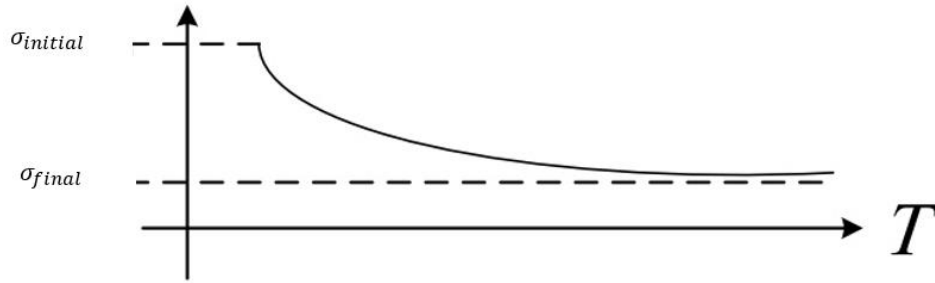

Fig. 3. Reduction of standard deviation over time.

The seed is scattered with a standard deviation according to the following formula:

$$\sigma_t = \left(\frac{T - t}{T} \right)^n [(\sigma_{initial} - \sigma_{final}) + \sigma_{final}]. \tag{15}$$

IV.   If the total number of plants reaches P_{max}, they are all sorted and the extra plants (with lower merit) are removed.

V.   If the termination conditions are not met, we return to step B, otherwise ends.

Thus, the optimization process of the invasive weed optimization algorithm starts with the creation of an initial population. Therefore, the most important part of any algorithm is the proper design of the initial solution. For example, *Fig. 4* shows the initial solution of the problem considering 4 buyers and 3 sellers in 2 different scenarios for 1 product.

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Scenario	Buyer			
	1	2	3	4
1	2	4	3	1
2	1	4	2	3

**Fig. 4. A representation of the initial
solution to the problem.**

Fig. 4 represents the initial solution to the problem. In this figure, a replacement of natural numbers into (buyer) numbers is considered for each scenario and each product. Therefore, it can be stated that the initial solution of the matrix is $|S| * |P| * |J|$.

The following steps should be performed to decode the solution of *Fig. 4*.

I.   According to the amount of uncertain demand of each buyer, first, the amount of orders of each buyer is determined based on *Eq. (5)*.

II.   The total demand and shortage is determined based on *Eq. (4)*.

III.   For each scenario and each product:

   − *The highest priority is selected from the numbers in Fig. 4.*
   − *The lowest ordering, holding, and shortage costs will be determined by the selected buyer/seller.*
   − *The amount of demand that can be transferred to the buyer is limited based on Eq. (3).*
   − *Each buyer's order quantity is updated.*
   − *If the buyer's entire demand is met, its priority is reduced to zero.*
   − *This process is repeated until all priorities are reduced to zero.*

IV.   If *Eq. (2)* and *Eqs. (6)-(8)* are not met, a penalty will be added to the objective function.

V.   The average total cost, the average cost in each scenario, and the minimum cost are calculated based on *Relations (9)-(11)*.

VI.   The value of the objective function of the problem is calculated by considering the penalty.

After presenting the initial solution to the problem and explaining the weed optimization algorithm, numerical examples in different sizes have been solved.

5|Analysis of Numerical Examples

In the present section, various numerical examples are considered to solve the resilient, sustainable supply chain model with an inventory management approach by the vendor. The invasive weed optimization algorithm and Baron's method have been used to solve these numerical examples. Due to the importance of setting the parameter of the meta-heuristic algorithm in increasing its efficiency in achieving near-optimal solutions, this operation has been done with the Taguchi method.

In the Taguchi method, three levels are proposed for each invasive weed optimization algorithm parameter. Each level contains parameter values that the correct combination of these levels increases the algorithm's efficiency in achieving a solution close to the optimum. The invasive weed optimization algorithm has 6 parameters; the suggested parameter value of each level is shown in *Table 1*. To achieve the best combination, Taguchi's method proposed a series of tests, as shown in *Table 2*. In *Table 2*, the unscaled value of each test is also shown in its last column.

Table 1. Suggested parameter values for each parameter setting level.

Parameter	Suggested Level Value		
	1	2	3
N0	100	150	200
Max it	200	300	400
S_{max}	5	6	7
S_{min}	0	1	2
n	1	2	3

Table 2. Taguchi tests for 5 parameters of the invasive weed optimization algorithm.

Test	N0	Max it	S_{max}	S_{min}	n	Unscaled result
1	1	1	1	1	1	0.053
2	1	1	1	1	2	0.078
3	1	1	1	1	3	0.028
4	1	2	2	2	1	0.103
5	1	2	2	2	2	0.097
6	1	2	2	2	3	0.189
7	1	3	3	3	1	0.127
8	1	3	3	3	2	0.112
9	1	3	3	3	3	0.074
10	2	1	2	3	1	0.009
11	2	1	2	3	2	0.196
12	2	1	2	3	3	0.063
13	2	2	3	1	1	0.187
14	2	2	3	1	2	0.092
15	2	2	3	1	3	0.084
16	2	3	1	2	1	0.133
17	2	3	1	2	2	0.105
18	2	3	1	2	3	0.199
19	3	1	3	2	1	0.025
20	3	1	3	2	2	0.063
21	3	1	3	2	3	0.173
22	3	2	1	3	1	0.082
23	3	2	1	3	2	0.059
24	3	2	1	3	3	0.190
25	3	3	2	1	1	0.012
26	3	3	2	1	2	0.020
27	3	3	2	1	3	0.000

Based on the results of the tests and the analysis of the mean S/N ratio diagram according to *Fig. 5*, the optimal levels of each parameter have been determined, and the optimal value of each factor has been obtained. Based on this analysis, the primary population (N0) is placed at the third level and equals 200. The maximum number of repetitions (Max it), the minimum number of seeds (Smin), and the nonlinear index (n) are placed in the first level and are equal to 200, 0 and 1, respectively. Finally, the maximum number of seeds (Smax) is placed in the second level and is equal to 6.
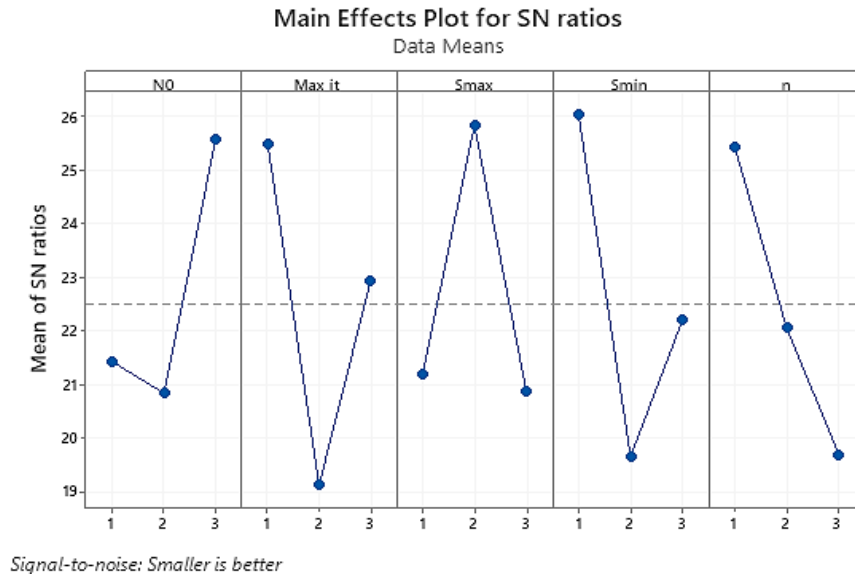
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Fig. 5. The mean of the S/N ratio diagram.

Various numerical examples have been solved based on the optimal parameters obtained from the weed optimization algorithm. The size of the designed numerical examples and the value of the mathematical model parameters are shown in *Tables 3* and *4*. The value of the parameters of the mathematical model is classified into two categories: deterministic and non-deterministic data. The data is in the form of a uniform distribution function for each fuzzy number.

Table 3. Size of numerical examples.

Numerical Examples	I	J	P	S	R	Numerical Examples	I	J	P	S	R
1	3	4	2	2	4	9	12	15	5	3	6
2	3	5	2	2	4	10	12	18	5	3	6
3	5	6	3	2	4	11	15	21	5	3	6
4	5	7	3	2	4	12	15	25	6	4	6
5	8	8	3	2	5	13	18	28	6	4	8
6	8	9	4	2	5	14	18	30	8	4	8
7	10	10	4	3	5	15	20	35	8	4	8
8	10	12	4	3	5	16	20	40	10	4	8

Table 3 shows 16 different numerical examples that were randomly generated.

Table 4. Values of mathematical model parameters.

Parameter	Mathematical Model Data	Parameter	Mathematical Model Data
fi_r	$\sim U(1000; 2000)$	ρ_{is}	50
f_p	$\sim U(0.2; 0.4)$	V	1
e_i	$\sim U(2; 4)$	T	5
φ_{is}	500	E	3
t_i	$\sim U(6; 10)$	F_r	$\sim U(40; 60)$
δ_{is}	1200	N	20
v_i	$\sim U(5; 10)$	γ	0.4
ω	2	θ	0.2

Parameter	Optimistically	Probable	Pessimistically
dem_{jps}	$\sim U(40; 60)$	$\sim U(60; 80)$	$\sim U(80; 100)$
ov_{ip}	$\sim U(70; 100)$	$\sim U(100; 120)$	$\sim U(120; 150)$
ob_{jp}	$\sim U(80; 110)$	$\sim U(110; 130)$	$\sim U(130; 160)$
h_{jp}	$\sim U(10; 12)$	$\sim U(12; 15)$	$\sim U(15; 18)$
s_{ip}	$\sim U(150; 200)$	$\sim U(200; 250)$	$\sim U(250; 300)$
k_{ip}	$\sim U(150; 200)$	$\sim U(200; 250)$	$\sim U(250; 300)$

To analyze and solve numerical examples, the uncertainty rate's value in each scenario is considered 0.5. Also, the probability of occurrence of each scenario is equal to $1/|S|$. The value of the objective function of each numerical example (total cost) is obtained by the weed optimization algorithm and the Baron method. To achieve the best output from the weed optimization algorithm, each numerical example is executed 3 times, and the lowest objective function value is recorded in *Table 5*. Also, the time needed to solve the problem using the two proposed methods is shown in *Table 5*.

Table 5. Outputs from solving different numerical examples.

Numerical Examples	Total Cost		Solution Time	
	IWO	Baron	IWO	Baron
1	234415.7	234122.2	13.56	86.59
2	257326.7	256836.8	15.81	297.66
3	281055.3	278043.1	20.77	697.18
4	305056.9	299674.2	26.05	1536.42
5	317696.3	-	31.73	2000<
6	321118.1	-	41.80	2000<
7	334817.5	-	53.61	2000<
8	340101.7	-	67.67	2000<
9	356026.1	-	87.24	2000<
10	386091.0	-	109.64	2000<
11	400884.7	-	134.58	2000<
12	411029.2	-	165.60	2000<
13	439669.6	-	206.28	2000<
14	447806.5	-	254.91	2000<
15	468489.3	-	309.44	2000<
16	488100.1	-	376.10	2000<

According to *Table 5*, Baron's exact method can only solve numerical examples of small size, and its solving time is recorded as 1536.42 seconds in the fourth numerical example. Meanwhile, the weed optimization algorithm solved the fourth numerical example in 26.05 seconds. Also, the maximum percentage of the relative difference between the invasive weed optimization algorithm and the Baron method equals 1.79%. This shows that the algorithm used to solve the problem is highly efficient in achieving a near-optimal solution quickly.

On the other hand, the above analysis shows that as the size of the numerical example increases, the time to solve the problem increases exponentially. The nonlinearity of the mathematical model and the exponential increase of the problem-solving time indicate that the problem is Np-hard. As a result, the resilient, sustainable supply chain model with an inventory management approach by the seller is one of the Np-Hard problems. *Fig. 3* shows the time to solve each numerical example with the two mentioned methods.

Considering the low relative difference percentage between the two solution methods, the convergence of numerical example number 1 with the invasive weed optimization algorithm has been shown in 200 consecutive iterations. In this numerical example, the total cost of the management system available by the seller by the Baron method was equal to 334122.2. According to the convergence of *Fig. 4*, the invasive weed optimization algorithm equals 334415.7.
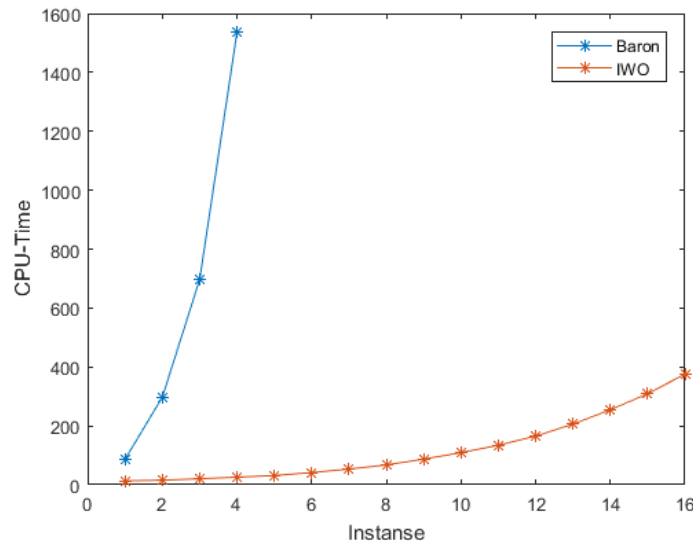
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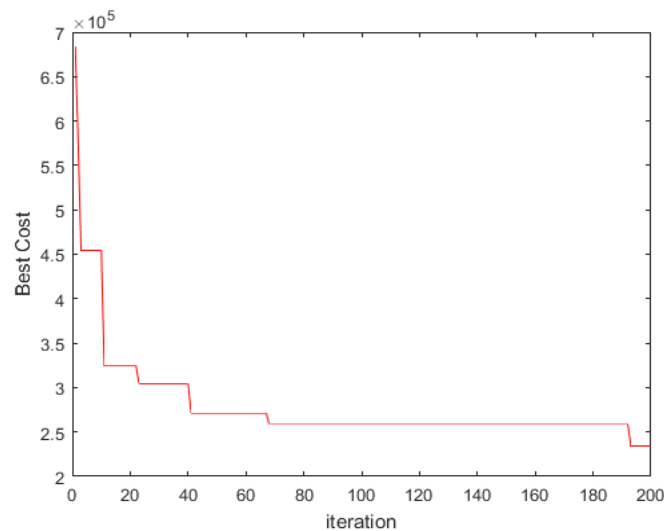**Fig. 3. Time to solve numerical examples in different sizes.**

Fig. 4. Convergence of the invasive weed optimization algorithm in achieving the optimal solution in numerical example number 1.

On the other hand, the weed optimization algorithm's speed of achieving the optimal solution in this numerical example was 6.4 times faster than the Baron method. Therefore, it can be shown that the efficiency of the weed optimization algorithm is very high in solving the inventory management model by the seller in the sustainable supply chain. With this solution, Baron's method was used to analyze the sensitivity and check the cost changes of the whole system under different conditions.

Table 6 examines the changes in the total cost of the inventory management system by the seller at different uncertainty rates. In system analysis, the value of the uncertainty rate in each scenario is considered equal to 0.5. If the uncertainty rate increases or decreases, the system's total cost is obtained as follows.

Changes in the uncertainty rate on the system's total cost show that with its increase, the total costs have increased due to the increase in the amount of demand in the system. As a result, with the increase in demand, the amount of orders issued to the seller has increased, and due to the increase in orders, maintenance, and even shortage costs, the costs of the entire inventory management system by the seller have increased. *Fig. 5* shows these changes in the uncertainty rate on total costs.

**Table 6. The total cost of the inventory management system
by the seller at different rates of uncertainty.**

Uncertainty Rate	Total Cost	Percentage of Changes
0.1	294735.2	-11.78%
0.2	302677.4	-9.41%
0.3	310268.4	-7.13%
0.4	325548.1	-2.56%
0.5	334122.2	0
0.6	347652.6	+4.04%
0.7	356748.6	+6.77%
0.8	368420.4	+10.26%
0.9	386174.6	+15.57%
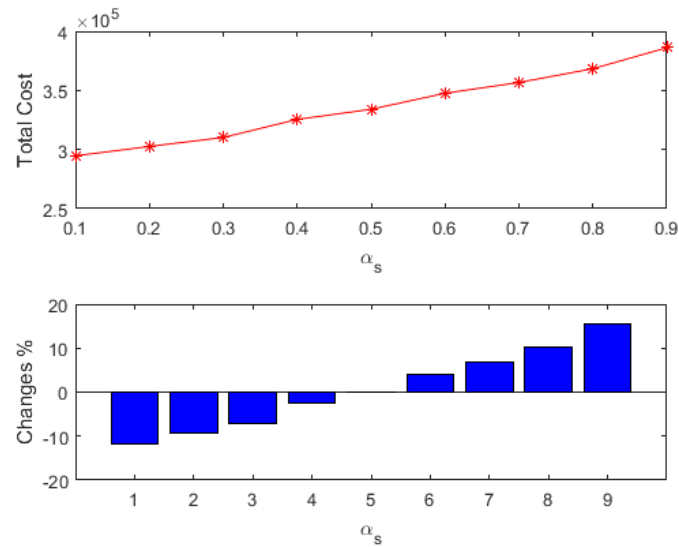
**Fig. 5. Cost changes of the total inventory system under
different uncertainty rates.**

On the other hand, in *Table 7*, the influence of the stability coefficient of the model on the costs of the whole system has been investigated. The stability coefficient of the model affects the maximum execution of buyers' orders.

Table 7. The total cost of the inventory management system by the seller in different stability coefficients.

Stability Factor	Total Cost	Percentage of Changes	The Total Amount of Demand Fulfillment
1	317286.4	-5.03%	76.25%
2	334122.2	0	82.48%
4	384896.5	15.19%	86.74%
6	431468.8	29.13%	93.11%
8	497358.2	48.85%	9578%
10	584635.1	74.85%	98.10%

Accordingly, it is found that with the increase in the stability factor of the model, the costs of the entire inventory management system have increased due to the increase in the number of orders and production. So, with a 5-fold increase in the stability coefficient, the demand value has increased from 82.48% to 98.10%. *Fig. 6* shows the change process of this analysis in different stability coefficients of the model.
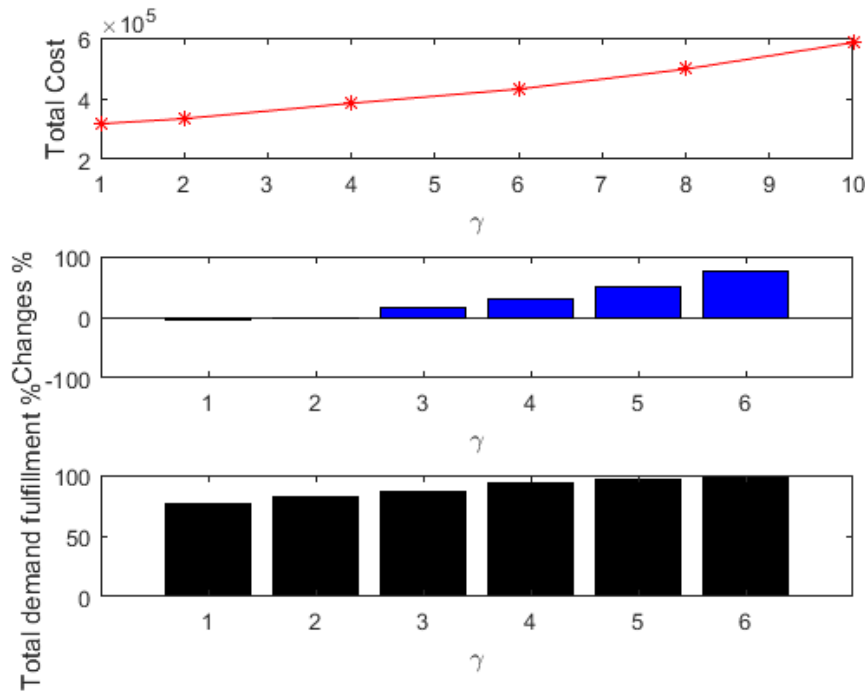
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**Fig. 6. Changes in the total cost of the inventory system in different
stability coefficients of the model.**

Finally, in the last sensitivity analysis, the seller investigated the resilience of the inventory management model. In this analysis, according to *Fig. 7*, it can be seen that by increasing the resilience factor, the amount of the order transferred to the buyer is reduced, leading to a reduction in ordering and maintenance costs.
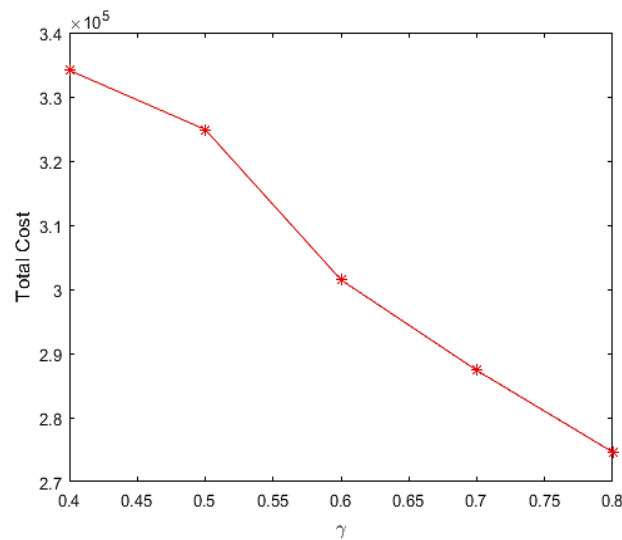

**Fig. 7. Changes in the total cost of the inventory system in
different resilience coefficients of the model.**

6 | Conclusion

This paper presents a new model of a resilient sustainable supply chain network under the uncertainty of demand and costs with the seller's inventory management approach. The designed model aimed to make optimal decisions regarding the number of soft transfer orders to customers and the allowed shortage. Optimizing the number of vehicles to transfer items to buyers was another decision in this model. The

ultimate goal of making the above decisions was to minimize the costs of the entire inventory management system by the seller, considering environmental and social constraints. The stable-fuzzy-probabilistic optimization method was used to control non-deterministic parameters and to solve the model, the invasive weed optimization algorithm of the Baron method was used.

By examining the results, it was observed that Baron's exact method was only able to solve numerical examples of small size, and its solving time was recorded as 1536.42 seconds in the fourth numerical example. Meanwhile, the weed optimization algorithm solved the fourth numerical example in 26.05 seconds. Also, the maximum percentage of the relative difference between the invasive weed optimization algorithm and the Baron method equals 1.79%. This shows that the algorithm used to solve the problem is highly efficient in achieving a near-optimal solution quickly.

Moreover, the changes in the uncertainty rate on the system's total cost showed that with its increase, the total costs have increased due to the increase in the amount of demand in the system. As a result, with the increase in demand, the amount of orders issued to the seller has increased, and due to the increase in orders, maintenance, and even shortage costs, the costs of the entire inventory management system by the seller have increased. The increase in the stability coefficient of the model has also increased the total costs of the inventory management system due to the increase in the number of orders and production. So, with a 5-fold increase in the stability coefficient, the demand value has increased from 82.48% to 98.10%.

The results of general calculations show that the seller's implementation of the inventory management approach greatly reduces the costs of the entire sustainable supply chain system and increases its productivity. For future research, multiple objective functions should be considered. It is also suggested that the Economic Production Quantity (EPQ) model be considered in the model. On the other hand, developing meta-credit algorithms can also be considered an effective solution to solving the problem.

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